# Nonlinear Cherenkov radiation at the interface of two different nonlinear media

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**Abstract:** We discuss the nonlinear response due to the spatial modulation of the second-order susceptibility at the interface between two nonlinear media, and experimentally demonstrate that the nonlinear Cherenkov radiation is enhanced by the interface of two nonlinear crystals with a large disparity in  $\chi^{(2)}$ . In our experiment, the intensity of the nonlinear Cherenkov radiation generated at the nonlinear interface was approximately 4 to 10 times that at the crystal boundary. This result suggests potential applications to efficient frequency conversion.

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#### **References and links**

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### 1. Introduction

Nonlinear Cherenkov radiation (NCR) [1], an extension of the Cherenkov radiation well known in particle physics to nonlinear optics, has attracted considerable interest in recent years. In NCR, when the phase velocity of the fundamental wave (FW) exceeds that of the second harmonic (SH) wave, the SH radiates at an angle  $\theta = \arccos(v_2/v_1)$ , where  $v_1$  and  $v_2$  denote the phase velocities of the FW and SH, respectively. This angle is determined by the dispersion relation of the nonlinear crystal and generally requires the crystal to be normally dispersive. Previous studies demonstrated that, under anomalous-like dispersion conditions, NCR can also be generated by modulating the phase velocity of the polarization wave [2]. NCR has been thoroughly researched in different nonlinear processes, such as second [3] and cascaded high-order harmonic generation [4–6], sum-frequency generation [7], and difference-frequency generation [8]. Such studies suggest potential applications to efficient frequency conversion [9], terahertz wave generation [10], ultrashort pulse-sharping [11], non-destructive domain-wall detection and imaging [12, 13], and so on.

Generally, efficient NCR occurs in nonlinear crystals with a random domain structure and in artificial periodic domain reversed structures [14, 15], as well as waveguide structures [16, 17]. The domain wall, which is the interface between antiparallel domains, modulates the second-order nonlinear coefficient  $\chi^{(2)}$  from 1 to -1. As NCR is a longitudinal auto-phase-matched process, domain structures can provide reciprocal vectors to compensate the phase mismatching in the transverse direction. Recent theoretical [18] and experimental [19] studies showed that NCR can be enhanced at crystal boundaries by a sharp spatial modulation of  $\chi^{(2)}$  from 1 to 0. However, for some nonlinear media, especially UV-transparent crystals, the intensity of NCR generated at the boundary is not very high since the nonlinear coefficient is comparatively small. The scenario where NCR is generated at an interface between two different quadratic crystals, where  $\chi^{(2)}$  modulates from  $C_1$  to  $C_2$  (constants), has not been fully discussed. Theoretically, if the vacuum on the one side of the boundary is substituted by a nonlinear crystal with a much higher  $\chi^{(2)}$  modulation.

In this letter, we fabricated a structure where two crystals with different nonlinear coefficients are tightly abutted upon each other, We then demonstrated, both theoretically and experimentally, that NCR in a bulk medium can be enhanced by the presence of another medium. As the

radiation angle is determined by the dispersion relationship of the medium, the added crystal does not change the NCR properties. We observed a four- to tenfold enhancement of NCR under normal dispersion conditions. In an anomalous dispersion environment, enhanced NCR was also recorded.

# 2. The coupled wave equation and simulation



Fig. 1. NCR Simulation at an interface. (a) Structure of a nonlinear medium. The y = 0 plane is the interface between media 1 and 2, which have different second nonlinear coefficients  $C_1\chi^{(2)}$  and  $C_2\chi^{(2)}$ , respectively. The FW is incident along the x axis. (b), (c) and (d) are the simulation results under the conditions  $C_1 = 1$ ,  $C_2 = 1$ ;  $C_1 = 1$ ,  $C_2 = 0$ ;  $C_1 = 1$ ,  $C_2 = 10$ , respectively.

To analyze the second harmonic (SH) generated at the nonlinear interface, we consider two semi-infinite media with different second-order nonlinear coefficients  $C_1\chi^{(2)}$  and  $C_2\chi^{(2)}$ , as in Fig. 1(a). For simplicity, the refractive index is assumed to be homogenous. The FW incident along the x axis is a Gaussian beam with width *a*. Considering the slowly varying envelope approximation, the coupled wave equation of SH is [20, 21]:

$$\left(\frac{\partial}{\partial x} + \frac{i}{2k_2}\frac{\partial^2}{\partial y^2}\right)A_2(x,y) = -i\frac{k_2}{2n_2^2}g(y)\chi^{(2)}(A_1e^{-y^2/a^2})^2e^{i\Delta kx},$$
(1)

where  $n_2$  is the refractive index of the SH,  $A_1$  and  $A_2$  denote the complex amplitudes of the FW and SH, respectively. The phase mis-matching is  $\Delta k = k_2 - 2k_1$ , where  $k_1$  and  $k_2$  are the wave vectors of the FW and SH, and g(y) is the spatial modulation function of  $\chi^{(2)}$  along the y axis:

$$g(y) = \begin{cases} C_1 & y \ge 0\\ C_2 & y < 0. \end{cases}$$
(2)

The FW amplitude is assumed to be constant. Using the Fourier transform, we can solve the coupled wave equation, and the intensity of SH  $I_2(x,k_y) = |A_2(x,k_y)|^2$  can then be expressed as:

$$I_{2}(x,k_{y}) = \left[\frac{k_{2}}{2n_{2}^{2}}\chi^{(2)}\right]^{2}I_{1}^{2}x^{2}sinc^{2}\left[\frac{(\Delta k - k_{y}/2k_{2})x}{2}\right] \\ \left|(C_{1} + C_{2})\sqrt{\frac{\pi}{8}}ae^{-\frac{a^{2}k_{y}^{2}}{8}} + i(C_{1} - C_{2})\frac{\sqrt{\pi}}{2}aD(\frac{ak_{y}}{2\sqrt{2}})\right|^{2},$$
(3)

where  $k_y = k_2 \sin\theta$  is the transverse wave vector of the SH,  $\theta$  is the radiation angle, and  $D(\frac{ak_y}{2\sqrt{2}})$  denotes the Dawson function.

Equation (3), indicates that, when  $C_1 - C_2 = 0$ , the SH intensity  $I_2$  gathers in the direction defined by  $k_y = 0$ , since  $e^{-a^2k_y^2/8}$  decreases sharply with increasing  $k_y$ . When  $C_1 - C_2 \neq 0$ , the radiation angle of the SH is determined by the term containing  $sinc^2[(\Delta k - k_y/2k_2)x/2]$ .

The solution of  $\Delta k - k_y/2k_2 = 0$  is  $k_y^2 = k_2^2 - (2k_1)^2$ , assuming  $k_y^2 \ll k_2^2$ . From the geometrical relationship, we can deduce the radiation angle satisfying  $k_2 \cos\theta = 2k_1$ , which is exactly the longitudinal phase-matching condition of NCR. Nonlinearity enhancement can be attributed mainly to the term of  $C_1 - C_2$ , whose absolute value is directly proportional to the SH intensity, according to Eq. (3). The SH intensity depends on the sharp change of  $\chi^{(2)}$ , i.e., if the absolute value of  $C_1 - C_2$  increases, the NCR intensity is enhanced.

The simulation results for different parameters are depicted in Figs. 1(b)–1(d). The FW wavelength is 800 nm. Both the FW and NCR are o-polarized. The interaction length is 200  $\mu m$ . Since we are concerned about the spatial modulation of  $\chi^{(2)}$  at the medium interface, other properties of media 1 and 2 are supposed to be the same in the simulation. For simplicity and without loss of generality, the dispersion of a  $\beta - BaB_2O_4$  crystal (BBO) is assumed for the media on both sides of the interface, unless otherwise stated. The simplest situation is to consider a whole bulk medium where  $C_1 = C_2$ . The result is a general phase-mismatched SH generation, with an extremely weak SH output that is collinear with the FW at zero angle, as shown in Fig. 1(b). In Fig. 1(c), the modulation function of  $\chi^{(2)}$  changes from 1 to 0. It describes NCR at the boundary of a nonlinear crystal. The radiation angle of the efficient SH is  $\pm 11.28^{\circ}$ , as is the calculated Cherenkov angle in BBO. Figure 1(d) shows the simulation result in agreement with our analysis where  $C_1 = 1$ ,  $C_2 = 10$ . The NCR intensity is a hundred times that in Fig. 1(c). Notably, the NCR radiation angle is the same as that in Fig. 1(c) since it depends solely on the dispersion relationship between the SH and FW.

# 3. Experiment results and discussion

In the experiment, two different nonlinear crystals tightly abutted upon each other under high pressure were used to achieve a sharp modulation of  $\chi^{(2)}$ . For Sample 1, as shown in Fig. 2(a), we chose a z-cut BBO crystal of size 5 mm(x)×10 mm(y)×10 mm(z) and a y-cut 5 mol% *MgO* : *LiNbO*<sub>3</sub> (LN) crystal of size 10 mm(x)×10 mm(y)×5 mm(z). The nonlinear coefficient of LN is much greater than that of BBO. And the orientation of the nonlinearity effects the polarization direction of nonlinear polarization wave and further determines the polarization direction of NCR. If the FW is o-polarized, the effective second-order nonlinear coefficient is *d*<sub>11</sub> in BBO, and *d*<sub>33</sub> in LN, and the absolute value of  $C_1 - C_2$  is about 14 [20]. For Sample 2 [Fig. 2(b)], the orientation of the LN crystal was changed to z-cut and its size was 5 mm(x)×10 mm(y)×10 mm(z). If the nonlinear process is oo-e, the effective second-order nonlinear coefficients in BBO and LN are both *d*<sub>31</sub>, and the absolute value of  $C_1 - C_2$  is about 48.



Fig. 2. Schematic experimental samples viewed from the top. (a) Sample 1: z-cut BBO and y-cut LN. (b) Sample 2: z-cut BBO and z-cut LN.

The phase-matching geometries in both scenarios are shown in Fig. 2. Since the two crystals are just pressed together, there is an air gap with a thickness of about several micron. The FW is incident from BBO to LN with an incident angle of  $\alpha$  with respect to the interface. Reflection

and refraction of the FW occur as a result of the refractive-index inhomogeneity. However this does not qualitatively affect NCR generation at the nonlinear interface emitting into the BBO side. The nonlinear polarization wave, stimulated by the obliquely incident FW, propagates along the interface. Its wave vector is  $k_{np} = 2k_1 \sin\alpha$ . When exceeding the threshold ( $k_{np} \le k_2$ ), Cherenkov radiation is emitted at an angle  $\theta = \arccos(k_{np}/k_2)$ . This remains unchanged, regardless of the presence of LN on the other side.

Figure 3(a) shows a schematic of our experiment. The FW was provided by a regenerative amplifier delivering 50 -fs pulses at a 1 -kHz repetition rate with a wavelength centering at 800 nm and an average power of 4.0 *mW*. The FW was loosely focused by a 100 -mm focal-length lens to a beam waist of 50  $\mu$ m. The beam was obliquely incident onto BBO crystal and reflected at the BBO-LN interface. The output light was then projected onto a screen.



Fig. 3. (a) Schematic experimental setup. (b) Internal NCR angle  $\theta$  under normal dispersion versus FW incident angle  $\alpha$ . Theoretical prediction (solid curve) and experimental results (symbols) are in good agreement. (c) Measured intensity and enhancement factor of SH generated at the BBO-LN interface and boundary of BBO crystal. (d), (f) and (h) are the recorded patterns with varying FW incident angle at the BBO-LN interface. (e), (g) and (i) are the recorded patterns of the FW incident on the BBO crystal boundary with the same angle as in (d), (f) and (h), respectively.

Sample 1 was first illuminated by the o-polarized FW. By turning the incident angle  $\alpha$ , opolarized second harmonic wave could be observed at different radiation angles. This is NCR in a normal dispersion environment, displayed as an oo-o nonlinear process in BBO. We measured the internal NCR angle  $\theta$  as a function of the FW incident angle  $\alpha$ . The theoretical prediction and experimental results are in good agreement, as shown in Fig. 3(b). Both at the boundary of the single BBO crystal and at the interface of Sample 1, the relationship between  $\theta$  and  $\alpha$ depends solely on the BBO dispersion.

The recorded patterns with different FW incident angles  $66.4^{\circ}$ ,  $63.8^{\circ}$ , and  $62.0^{\circ}$  are presented in Figs. 3(d), 3(f), and 3(h), respectively. For comparison, the NCR generated at the boundary of the single BBO crystal was also observed [see Figs. 3(e), 3(g), and 3(i)]. Obviously, at the same FW reflection angle, the intensity of NCR generated at the interface of Sample 1 was much higher than that generated at the BBO boundary. This observation verifies the preceding simulation result that NCR is enhanced at an interface where the modulation of  $\chi^{(2)}$  is sufficiently large. The enhancement factor ranges from approximately 4 [Fig. 3(d)] to 10 [Fig. 3(h)]

and varies with the FW reflection angle [Fig. 3(c)]. The quality of the interface, and the partial reflection of the FW from it, degrade the conversion efficiency as the two crystals are simply pressed together to form the interface and an air gap between them is unavoidable. Using diffusion bonding [22] or other direct bonding techniques, the interface of the two nonlinear materials may be much smoothed and the NCR intensity can be further enhanced.

Sample 2 was used to repeat the experiment. Compared with Sample 1, the o-polarized NCR was not enhanced because of the absence of  $d_{11}$  in LN. However, an e-polarized SH can be observed if the incident angle  $\alpha$  is smaller than 70° [Figs. 4(b)-4(e)]. This oo-e process occurs under anomalous-like dispersion conditions, considering the refractive index of BBO. When the SH wave vector  $k_{2e} \geq 2k_{1o}sin\alpha$ , the NCR condition is satisfied. The experiment results and theoretical prediction are also in good agreement [Fig. 4(a)]. This phenomenon was not observed in Sample 1 or in single BBO, because  $d_{31}$  is much smaller than  $d_{11}$  in BBO and the  $d_{13}$  of LN is zero. In an anomalous-like dispersion environment, the SH intensity may be further enhanced by using other crystals with a high nonlinear coefficient.



Fig. 4. (a) The internal NCR angle  $\theta$  under anomalous dispersion versus the incident angle of FW  $\alpha$ . (b)-(e) are the recorded patterns varying with the incident angle of FW.

The advantage of this method is that another material can be used to enhance the nonlinear process in the crystal of interest, without changing its properties. For example, in an LN crystal, light with wavelength below 380 nm is absorbed, but can be used to enhance ultraviolet SH generation in BBO. Our scheme may provide a new way for practical applications to ultraviolet laser sources and efficient nonlinear processes.

## 4. Conclusion

In summary, we demonstrated both theoretically and experimentally, that NCR generation can be enhanced at the nonlinear interface of two different nonlinear materials. The sharper the change in nonlinear coefficient, the higher the NCR intensity. Under normal dispersion conditions, the NCR generated at a BBO-LN interface is enhanced by a factor in the range of 4 to 10, compared with that at the boundary of a single BBO crystal. Under anomalous dispersion conditions, an oo-e type of NCR was observed as a result of the enhancement at the nonlinear interface. The added nonlinear material does not change the properties of the Cherenkov radiation, such as the emission angle.

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